A symbiosis of constraint optimization, symmetries and symmetry breaking for scalable Cloud deployment problems

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Joint work with Bogdan David, Flavia Micota and Daniela Zaharie

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Outline

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Benefits of optimal deployment:

1. the synthesis of deployment plans that are optimal by design
2. the integration of such deployment plans into the application modeling process, enables formal reasoning on a model of the deployed application.

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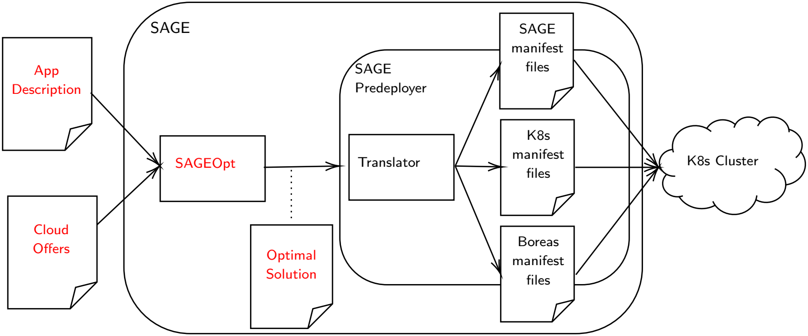

Figure: SAGE General Architecture

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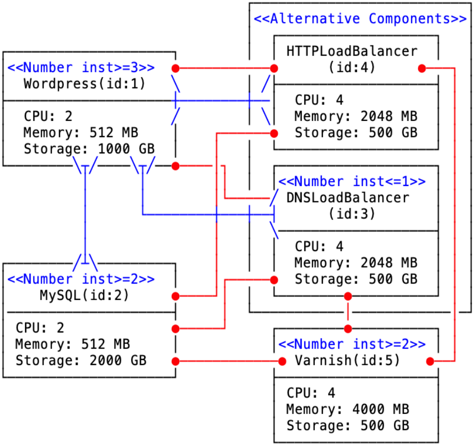

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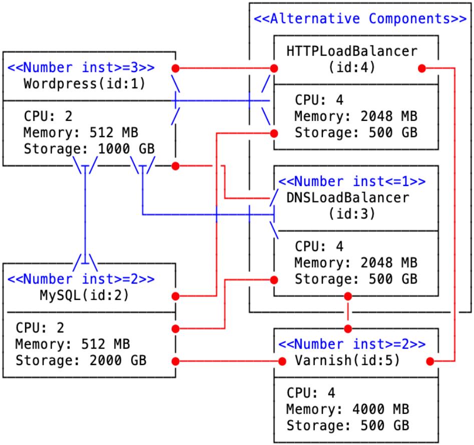

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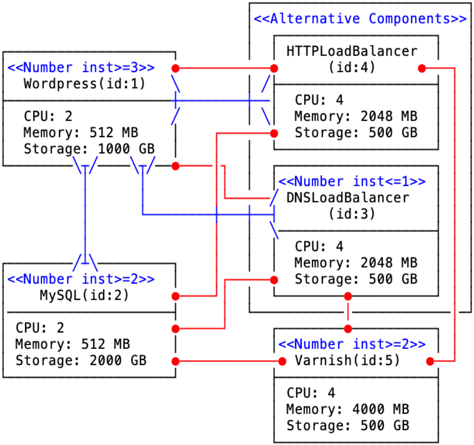

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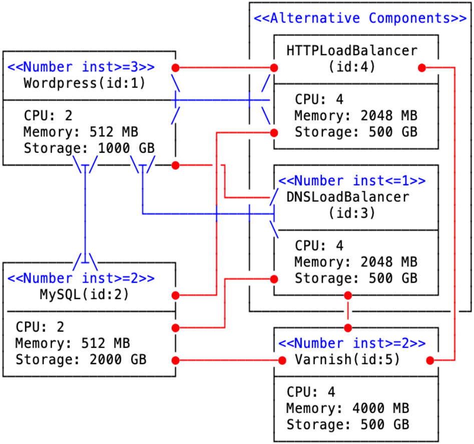

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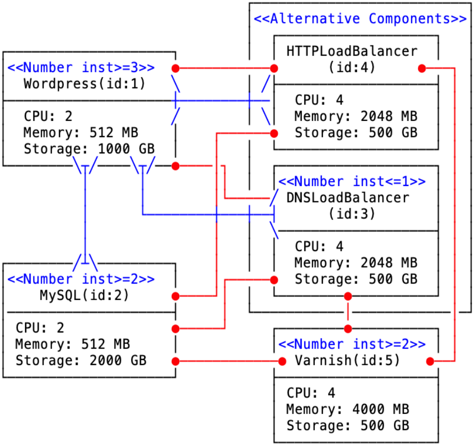

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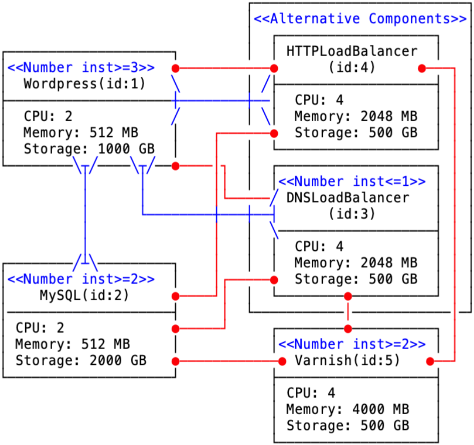

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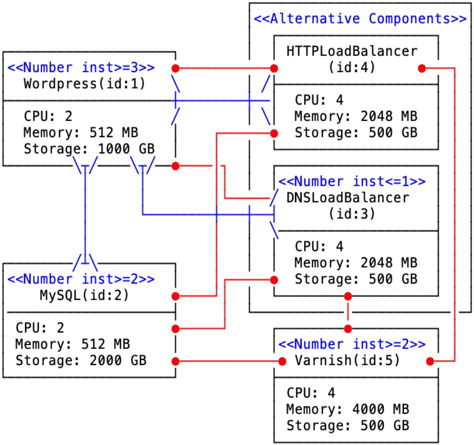

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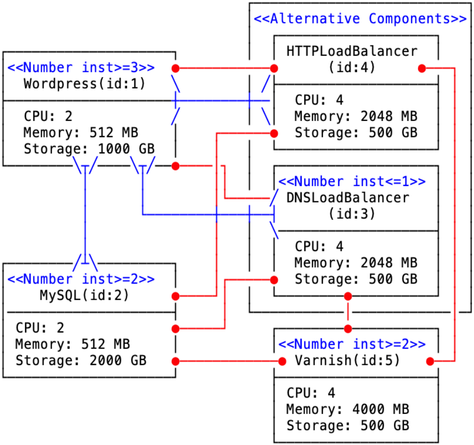

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  $\sum_{k=1}^{M} a_{3k} \leq 7 \sum_{k=1}^{M} a_{1k}, \ \ k = \overline{1, M}$
- Only one type of balancer must be deployed (*Exclusive deployment constraint*).
  $a_{3k} + a_{4k} \ \leq \ 1, \ \ k = \overline{1, M}$
- Components are characterized in terms of their resource demand (i.e. in terms of CPU cores, RAM and storage capacity).
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Cloud provider offers

Spot Instance Prices

Spot Instances | Defined Duration for Linux | Defined Duration for Windows

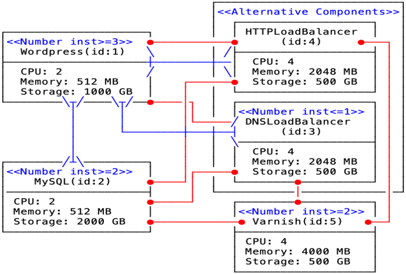
Region: EU (Ireland)

	Linux/UNIX Usage	Windows Usage
General Purpose - Current Generation		
t2.micro	$0.0038 per Hour	$0.0084 per Hour
t2.small	$0.0075 per Hour	$0.0165 per Hour
t2.medium	$0.015 per Hour	$0.033 per Hour
t2.large	$0.0302 per Hour	$0.0582 per Hour
t2.xlarge	$0.0605 per Hour	$0.1015 per Hour
t2.2xlarge	$0.121 per Hour	$0.183 per Hour
m3.medium	$0.0073 per Hour	$0.0633 per Hour
m3.large	$0.0306 per Hour	$0.1226 per Hour
m3.xlarge	$0.0612 per Hour	$0.2452 per Hour

Model	vCPU	CPU Credits / hour	Mem (GiB)	Storage
t2.nano	1	3	0.5	EBS-Only
t2.micro	1	6	1	EBS-Only
t2.small	1	12	2	EBS-Only
t2.medium	2	24	4	EBS-Only
t2.large	2	36	8	EBS-Only
t2.xlarge	4	54	16	EBS-Only
t2.2xlarge	8	81	32	EBS-Only

Remark: [snapshot from https://aws.amazon.com/ec2/] tens of thousands of price offers corresponding to different configurations and zones

Wordpress: Example Solution



- VM_1 (CPU:4, RAM: 30.5 GB, Storage: 1000 GB, Price: 0.0379 \$/hour): Wordpress+MySQL
- VM_2 (CPU:4, RAM: 30.5 GB, Storage: 1000 GB, Price: 0.0379 \$/hour): Wordpress+MySQL
- VM_3 (CPU:4, RAM: 7.5 GB, Storage: 2000 GB, Price: 0.021 \$/hour): Varnish
- VM_4 (CPU:4, RAM: 7.5 GB, Storage: 2000 GB, Price: 0.021 \$/hour): Varnish
- VM_5 (CPU:4, RAM: 7.5 GB, Storage: 2000 GB, Price: 0.021 \$/hour): HTTPLoadBalancer
- VM_6 (CPU:4, RAM: 7.5 GB, Storage: 2000 GB, Price: 0.021 \$/hour): Wordpress

Contents

Problem Formalization

General constraints

$$\begin{array}{rll}
\textit{Basic allocation} & \sum_{k=1}^{M} a_{ik} \geq 1 & \forall i = \overline{1, N} \\
\textit{Occupancy} & \sum_{i=1}^{N} a_{ik} \geq 1 \Rightarrow v_k = 1 & \forall k = \overline{1, M} \\
\textit{Capacity} & \sum_{i=1}^{N} a_{ik} \cdot R_i^h \leq F_{t_k}^h & \forall k = \overline{1, M}, \forall h = \overline{1, H} \\
\textit{Link} & v_k{=}1 \wedge t_k{=}o \Rightarrow \bigwedge_{h=1}^{H} \left(r_k^h{=}F_{t_k}^h\right) \wedge p_k{=}P_{t_k} & \forall o = \overline{1, O},\ O \in \mathbb{N}^* \\
 & \sum_{i=1}^{N} a_{ik} = 0 \Rightarrow t_k = 0 & \forall k = \overline{1, M}
\end{array}$$

where:

- $R_i^h \in \mathbb{N}^*$ is the hardware requirement of type h of the component i;
- $F_{t_k}^h \in \mathbb{N}^*$ is the hardware characteristic h of the VM of type t_k.

Problem Formalization (cont'd)

Application-specific constraints

Conflicts	$a_{ik} + a_{jk} \leq 1$	$\forall k = \overline{1, M},\ \forall (i,j)\ \mathcal{R}_{ij} = 1$
Co-location	$a_{ik} = a_{jk}$	$\forall k = \overline{1, M},\ \forall (i,j)\ \mathcal{D}_{ij} = 1$
Exclusive	*deployment*	
	$\mathcal{H}\left(\sum_{k=1}^{M} a_{i_1 k}\right) + \ldots + \mathcal{H}\left(\sum_{k=1}^{M} a_{i_q k}\right) = 1$	for fixed $q \in \{1, \ldots, N\}$
		$\mathcal{H}(u) = \begin{cases} 1 & u > 0 \\ 0 & u = 0 \end{cases}$
Require-	*Provide*	
	$n_{ij} \sum_{k=1}^{M} a_{ik} \leq m_{ij} \sum_{k=1}^{M} a_{jk}$	$\forall (i,j) \mathcal{Q}_{ij}(n_{ij}, m_{ij}) = 1$
	$0 \leq n \sum_{k=1}^{M} a_{jk} - \sum_{k=1}^{M} a_{ik} < n$	$n, n_{ij}, m_{ij} \in \mathbb{N}^*$

where:

- $\mathcal{R}_{ij} = 1$ if components i and j are in conflict (can not be placed in the same VM);
- $\mathcal{D}_{ij} = 1$ if components i and j must be co-located (must be placed in the same VM);
- $\mathcal{Q}_{ij}(n, m)$=1 if C_i requires at least n instances of C_j and C_j can serve at most m instances of C_i

Problem Formalization (cont'd)

Application-specific constraints

Full deployment $\sum_{k=1}^{M}\left(a_{ik} + \mathcal{H}\left(\sum_{j,\mathcal{R}_{ij}=1} a_{jk}\right)\right) = \sum_{k=1}^{M} v_k$

Deployment with bounded number of instances

$\sum_{i\in\overline{C}} \sum_{k=1}^{M} a_{ik}\langle\text{op}\rangle n$ $\quad |\overline{C}| \leq N,\ \langle\text{op}\rangle \in \{=,\leq,\geq\},\ n\in\mathbb{N}$

Find:

- assignment matrix a with binary entries $a_{ik} \in \{0,1\}$ for $i = \overline{1,N}$, $k = \overline{1,M}$, which are interpreted as follows:

$$a_{ik} = \begin{cases} 1 & \text{if } C_i \text{ is assigned to } V_k \\ 0 & \text{if } C_i \text{ is not assigned to } V_k. \end{cases}$$

- the type selection vector t with integer entries t_k for $k = \overline{1,M}$, representing the type (from a predefined set) of each VM leased.

Such that: the leasing price is minimal $\sum_{k=1}^{M} v_k \cdot p_k$

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- NP-hard

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In this presentation: Speeding-up exact methods by symmetry breaking

Case Study: Wordpress (cont'd)

Solution:

- *assignment matrix* with elements $a_{ij} \in \{0, 1\}$

$$a_{ik} = \begin{cases} 1 & \text{if component } C_i \text{ is assigned to machine} V_k \\ 0 & \text{if component } C_i \text{ is not assigned to machine} V_k. \end{cases}$$

- *type selection vector t* with elements $t_k \in \mathbb{N}$ ($k = \overline{1, M}$) representing the type (from a predefined set) of each VM leased.

both fulfilling the application constraints and minimizing the leasing price.

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For the case when the number of Wordpress instances is 3, we have:

- the assignment matrix

	V_1	V_2	V_3	V_4	V_5	V_6
$Wordpress(C_1)$	1	1	0	0	0	1
$MySql(C_2)$	1	1	0	0	0	0
$DNSLoadBalancer(C_3)$	0	0	0	0	0	0
$HTTPLoadBalancer(C_4)$	0	0	0	0	1	0
$Varnish(C_5)$	0	0	1	1	0	0

- the type vector: $t = [186, 186, 182, 182, 182, 182]$.

Contents

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#ins	#ofr=20	#ofr=40	#ofr=250	#ofr=500
		Chuffed		
3	2.13	4.18	57.72	458.21
4	26.03	114.18	1974.16	-
5	638.26	2230.39	-	-
		OR-Tools		
3	3.52	8.42	96.38	191.38
4	23.25	56.47	502.71	988.33
5	149.47	428.98	-	-
6	494.46	1174.36	-	-
		IBM CPLEX		
3	9.81	-	-	-
4	124.68	-	-	-
5	452.32	-	-	-
6	737.89	-	-	-
		Z3		
3	2.92	4.13	115.36	391.87
4	46.46	366.24	-	-

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Partial row/column symmetry are more often encountered in Cloud deployment problems.

Symmetry Breaking: Column Symmetries

Ordering decreasing

- (L) the columns by the number of components for columns representing VMs of the same type:

$$\sum_{i=1}^{N} a_{ik} \geq \sum_{i=1}^{N} a_{i(k+1)}, \quad \forall k = \overline{1, N-1}$$

- (LX) the columns by lexicographic order for columns representing VMs of the same type

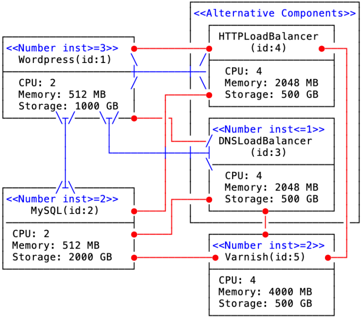
$$a_{\star k} \succ_{lex} a_{\star(k+1)}, \text{ where } a_{\star k} \text{ denotes the column } k.$$

- (PR) ordering decreasing the VMs by their characteristics (price, CPU, memory, storage)

$$P_1 \geq P_2 \geq ... \geq P_N, \quad \forall k = \overline{1, N}$$

Symmetry Breaking: Row Symmetries

(FV) pre-assigning, on separate VMs, the components composing the clique with maximum deployment size obtained from the conflict graph, i.e. the graph where the component instances are the nodes and the conflicts are the edges.

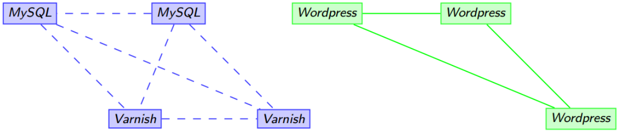


Example (**FV**: Wordpress with 3 Wordpress instances)

There are 3 cliques with maximum deployment size 4. Pick one:

- $[2 MySQL, 2 Varnish]$
- ~~$[3 Wordpress, 1 HTTPLoadBalancer]$~~
- ~~$[3 Wordpress, 1 DNSLoadBalancer]$~~

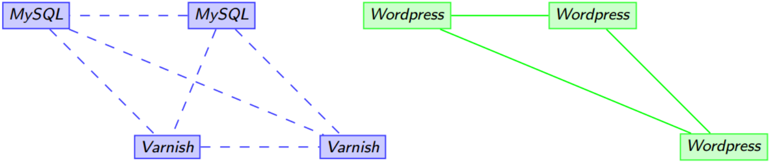
Examples of cliques



Symmetry Breaking: Row Symmetries (cont'd)

Example (**FV**: Wordpress with 3 Wordpress instances)

Clique with maximum deployment size 4: $[2MySQL, 2Varnish]$



	V_1	V_2	V_3	V_4	V_5	V_6
C_1	?	?	?	?	?	?
C_2	1	1	0	0	?	?
C_3	?	?	?	?	?	?
C_4	?	?	?	?	?	?
C_5	0	0	1	1	?	?

Symmetry Breaking: Finite combination of row and column symmetries

- FV, PR, L, LX,
- FVPR, FVL, FVLX, PRL, PRLX, LPR, LLX,
- FVPRL, FVPRLX, FVLPR, FVLLX, PRLLX, LPRLX,
- FVPRLLX, FVLPRLX

Example (PRLX (Wordpress with 3 Wordpress instances))

The assignment matrix:

	V_1	V_2	V_3	V_4	V_5	V_6
C_1	1	1	1	0	0	0
C_2	1	1	0	0	0	0
C_3	0	0	0	0	0	0
C_4	0	0	0	1	0	0
C_5	0	0	0	0	1	1

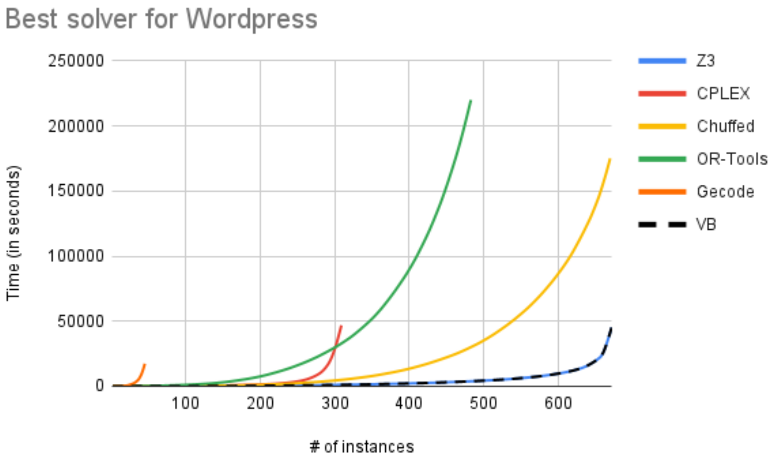
The price vector: $p = [379, 379, 210, 210, 210, 210]$.

Symmetry breakers:

$$P_1 \geq P_2 \wedge$$
$$P_1 = P_2 \Rightarrow a_{11} \geq a_{12} \wedge$$
$$P_1 = P_2 \wedge a_{11} \geq a_{12} \Rightarrow a_{21} \geq a_{22} \wedge$$
$$P_1 = P_2 \wedge a_{11} = a_{12} \Rightarrow a_{31} = a_{32} \wedge$$
$$P_2 \geq P_3 \ \wedge \ldots$$

Contents

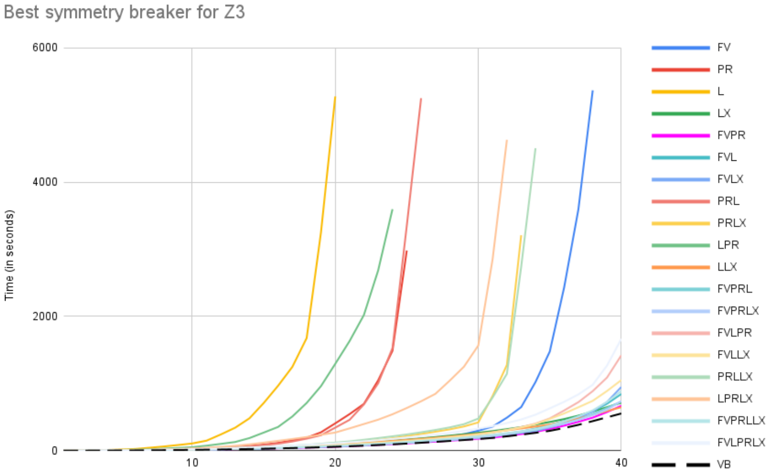
Experimental Results II



Symmetry Breaking: Row-Column Symmetries (cont'd)

Best symmetry breaker for Z3: FVPR

Remark: Combination of more than two symmetry breakers did not lead to better results although more symmetries are broken. This means that breaking more symmetries does not necessarily mean a computational improvement, since more more constraints are added.



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- Using symmetry breakers, we were able to solve use cases with up to 30 component instances and 500 Cloud offers.
- *Observation*: Number of variables and clauses is not really relevant as they can actually easy the problem to be solved.

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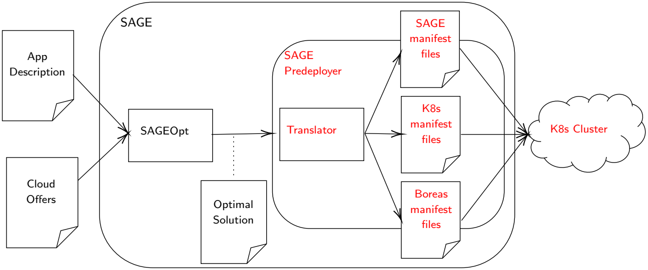

Figure: SAGE General Architecture

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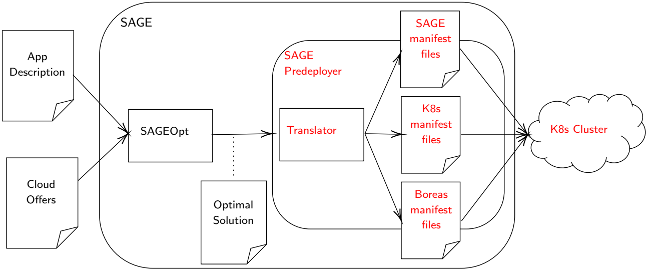

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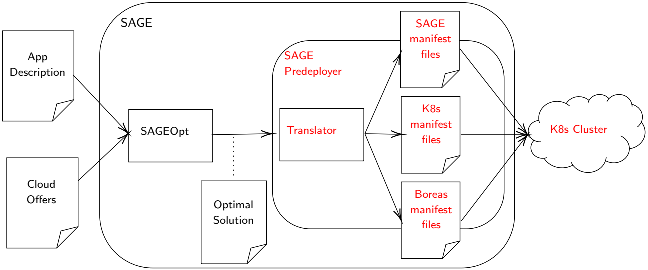

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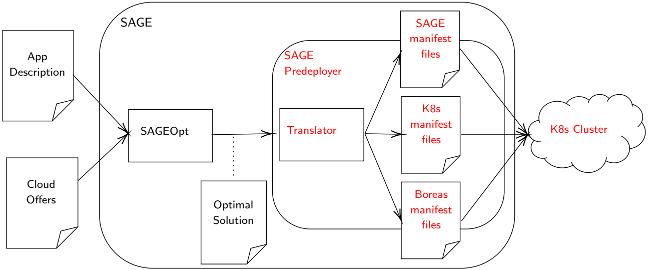

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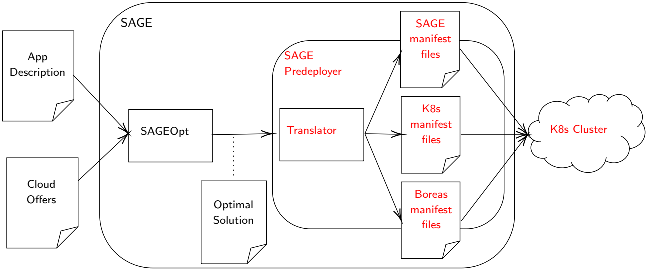

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